Dating market, familiarity graphs, and selectivity

Game Theory and Management, St.Petersburg

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The question: how do dating technologies affect the outcome?

The world is experiencing a change in search&dating technologies

- costs are severely reduced
- number of potential partners is almost unbounded
- 75M people use Tinder every month

Simultaneously, we face new demographic phenomena

- 41% US women of 25-44y are single and childless (Morgan Stanley)

Can it be an "evolutionary mismatch/trap"?

- our preferences developed over generations
- same as search and mating strategies
- rapid change in one without another might be problematic

Result: lower costs and wider menus may leave fewer people matched.

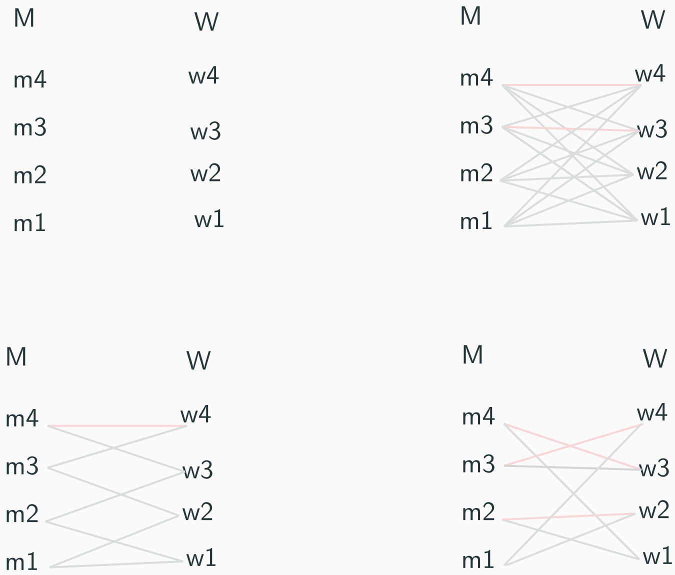
The results are based on two assumptions about preferences:

1. preferences are correlated
2. preferences are selective

Background

- Two-sided search and matching with frictions
  - block-segregation result: endogenous partitioning of types [McNamara and Collins, 1990, Eeckhout, 1999]
  - when agents obtain imperfect feedback, reducing costs can lead to infinite search [Antler and Bachi, 2022]
- Women are more selective than men [Fisman et al., 2006, Kelley and Malouf, 2013] *"supported by self-report surveys, speed-dating studies, analysis of on-line and newspaper personal ads, and laboratory analog studies "*
- Stable matching: marriage market and college admissions [Gale and Shapley, 1962]
  - Lower search costs increase social integration [Ortega and Hergovich, 2017]

Example for $n = 4$ men/women, familiarity degrees $k = 4, 2$, selectivity $s = 1/2$



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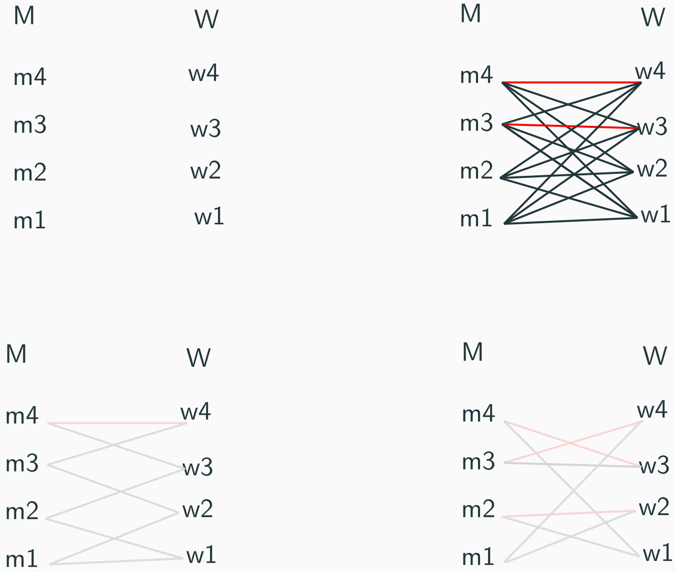

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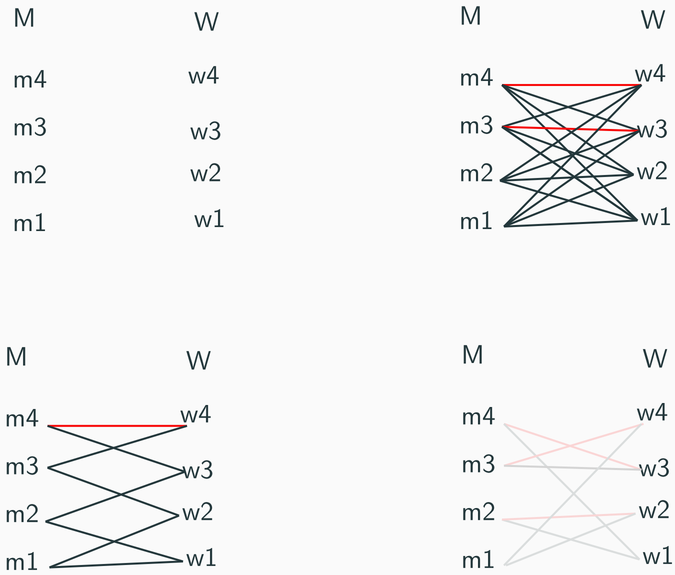

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Model: marriage market with familiarity graph

- set of n men M and set of n women W
- familiarity graph F prescribes who knows whom
  familiarity is always mutual: $w \in F(m) \iff m \in F(w)$
- regularity: each m knows k women, each w knows k men

Correlated preferences: agents agree on ranking of those that they know

- let each woman w have the same ranking $\bar{P}_w$ over M
- (let each man m have the same ranking $\bar{P}_m$ over W) - not needed
- Preferences P are derived from $\bar{P}$ restricted by graph F:
  for each $x \in M \cup W$, $P_x = \bar{P}_x|_{F(x)}$

Selective preferences: only a constant share is acceptable

- each m finds $s_m k$ women acceptable, $|w : wP_m m| = s_m k$
- each w finds $s_w k$ men acceptable, $|m : mP_w w| = s_w k$

Stable matchings

Our solution concept is stability:
- nobody is matched with unacceptable partner,
- no blocking pair (m, w) that prefers to be together but is not

- Stable matching always exists [Gale and Shapley, 1962]
- If preferences are homogeneous for one side, stable matching is unique
- What is the size of this stable matching?
- In the example with $n = 4, k = 2$, and $s_m = s_w = 0.5$: depending on the graph, $|\mu|$ varies from 1 to 3
- What is the average size of a stable matching for a random graph?

Average size of stable matching

As the choice set increases, there are two effects on the matching size:

- positive: each agent knows more acceptable partners
- negative: each non-top-ns agent has lower chances
  (in a complete graph only the top ns agents are matched)
- the two effect balance each other

Consider the case $s_m = s_w = s$. Define sparse graph:

- each agent knows exactly 1 acceptable partner, $k = 1/s$.

Calculate the size of stable matchings for random sparse graph.

Equal selectivity

Proposition 1: If $s_m = s_w = s$, then stable matchings for sparse graph, $k = 1/s$ and complete graph, $k = n$ have the same average size.

Proof. Let index denote preference, m_n is most preferred, m_1 is least preferred.

$$\langle|\mu^{k=1/s}|\rangle = \sum_{m=1}^{n} Prob(m\ is\ matched) =$$

$$\sum_{m=1}^{n} Prob(his\ acc.woman\ finds\ m\ acceptable) = \sum_{m=1}^{n} \frac{C_{m-1}^{k-1}}{C_{n-1}^{k-1}} =$$

$$\sum_{m=1}^{n} \prod_{i=1}^{k-1} \frac{m-i}{n-i}$$

Result 1

$$\sum_{m=1}^{n}\prod_{i=1}^{k-1}\frac{m-i}{n-i}=$$

$$\frac{(k-1)(k-2)\dots 2\cdot 1+k(k-1)\dots 3\cdot 2+(k+1)\dots 4\cdot 3+\dots}{(n-1)(n-2)\dots(n-k+1)}=$$

$$=\frac{(k+1)(k-1)\dots 3\cdot 2+(k+1)k(k-1)\dots 4\cdot 3+\dots}{(n-1)(n-2)\dots(n-k+1)}=$$

$$=\frac{n(n-1)(n-2)\dots(n-k+1)}{k(n-1)(n-2)\dots(n-k+1)}=\frac{n}{k}=sn=|\mu^n|$$

Intermediate case

- So far we know about two extremes: for sparse and complete graphs

$$\langle|\mu|\rangle = sn$$

- What can be said about the intermediate case?
- (m, w) is **mutual-best** if they are most preferred familiar partners: $\forall m' \in F(w) \setminus \{m\}, w' \in F(m) \setminus \{w\}$ we have $mP_w m'$ and $wP_m w'$

Corollary: For $k > 1/s$, average number of mutual-best pairs is n/k.

- remove mutual-best agents, then on average we have:
  - $n - n/k$ remaining agents
  - $k - 1$ familiar agents
- the ratio is the same $(n - n/k)/(k - 1) = n/k$
- at best, we can repeat it sk times: $sk * n/k = sn$

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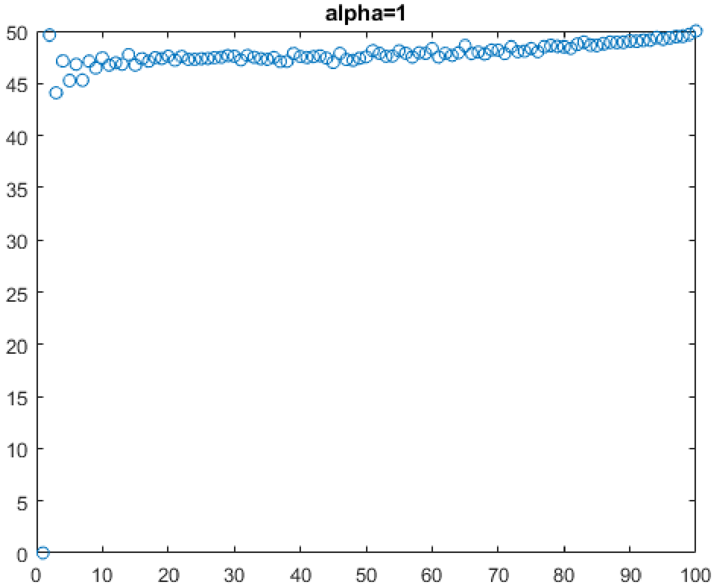
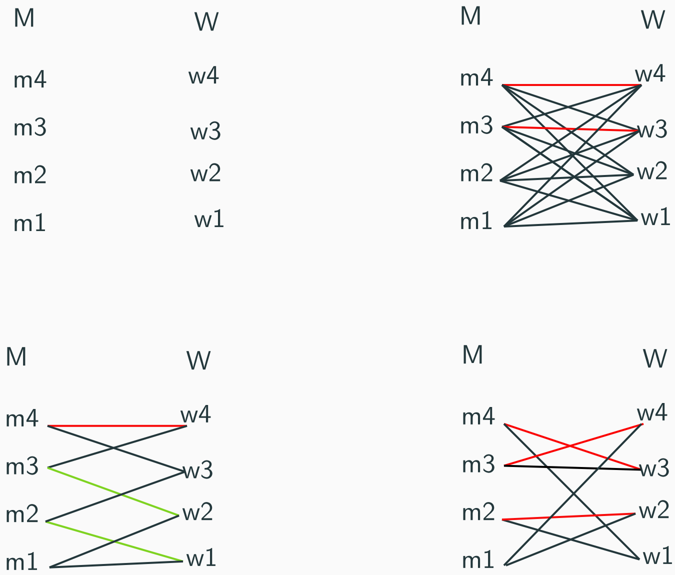
Simulations: $n = 100, s_m = s_w = 0.5$



Same example for $n = 4$ men/women, familiarity degrees $k = 4, 2$, selectivities $s_m = 1, s_w = 1/2$



Result 2

Let $s_m > s_w$, and consider a sparse graph for women: each woman knows 1 acceptable man (while each man knows $ks_m \geq 1$ acceptable woman).

Proposition 2: If $s_m > s_w$, then stable matchings for sparse graph has higher average size than for complete graph.

$$\langle|\mu^{k=1/s_w}|\rangle = n - \sum_{m=1}^{n} \left(1 - \prod_{i=1}^{k-1} \frac{m-i}{n-i}\right)^{s_m k} \geq ns_w$$

Proof. Consider some man m and his best acceptable woman w. He has same probability of being acceptable for w as before. But now m might have a second chance with his second best acceptable woman; and so on.

Empirical simulations show that $\langle|\mu^k|\rangle$ monotonically decreases from $k = 1/s_w$ to $k = n$.

Variable selectivity

Selectivity might depend on the size of the choice set.

In a field experiment, [Fisman et al., 2006] find that in small groups $s_m = s_w = s$, and that $s_w(k)$ decreases with k.

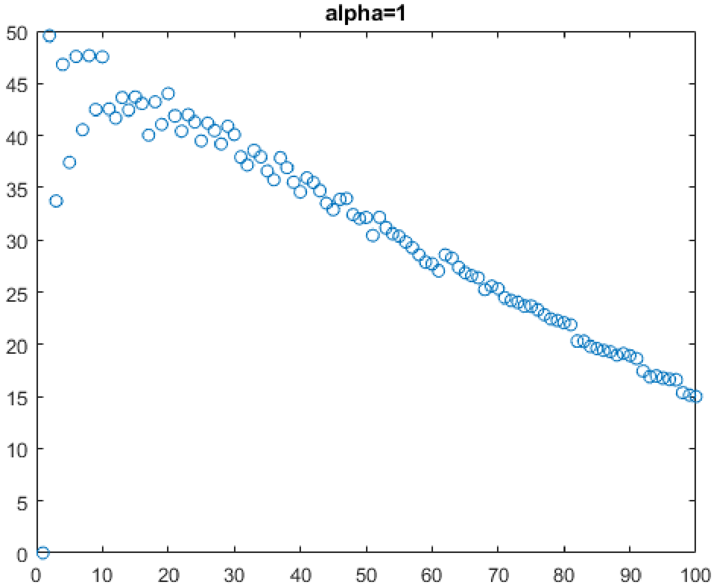
With this assumption, sparse graph gives ns_m pairs, while complete graph gives ns_w pairs.

Consider simulations with $n = 100$

men and women have homogeneous preferences

As k grows from 2 to 100, s_w linearly decreases from 0.5 to 0.15

Simulations: $s_m = 0.5$, s_w decreases from 0.5 to 0.15



Simulations for general case

Let each agent x's utility $u_x(y)$ from matching with partner y have three random components distributed uniformly on $[0, 1]$:

- the common component of partner y denoted as v_y^{common} ,
- the random idiosyncratic component $v_{xy}^{idiosyncratic}$,
- and the mutual preference component, $v_{xy}^{mutual} = v_{yx}^{mutual}$

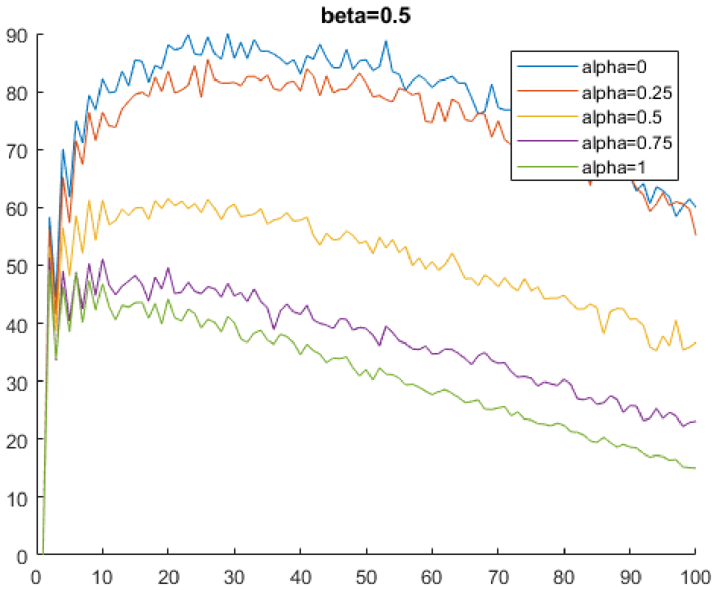
The total utility is their weighted sum:

$$u_x(y) = \alpha v_y^{common} + (1 - \alpha)(\beta v_{xy}^{random} + (1 - \beta) v_{xy}^{mutual}).$$

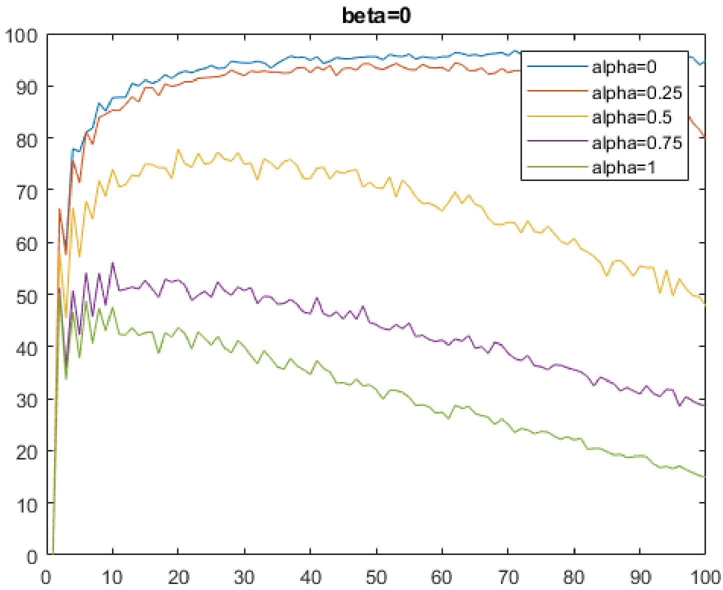
$n = 100$, $k = 1, \ldots, n$ # of simulations $mc = 10$
Preview of results:

- equal selectivity: sparse=full, intermediate slightly worse
- for general preferences: higher k – larger matching
- different selectivity: size for sparse graph twice larger
- for general preferences: intermediate optimum
- varying selectivity: stronger effect

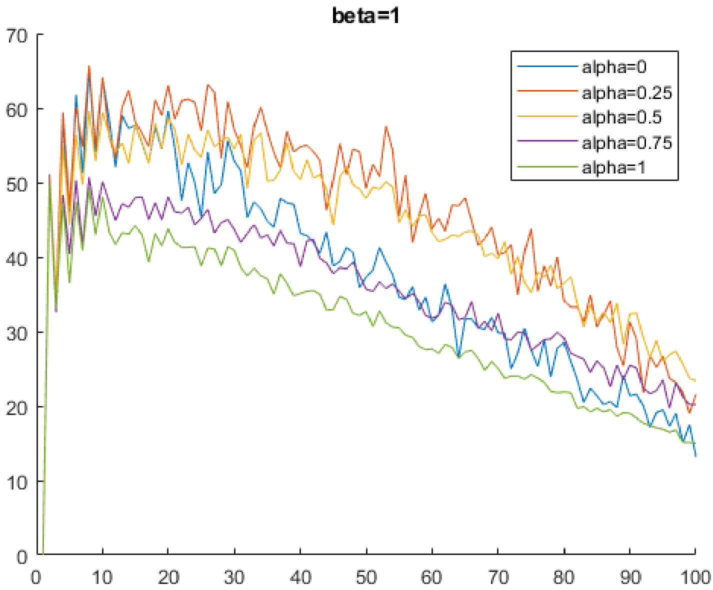
Simulations: $\beta = 0.5$, $s_m = 0.5$, s_w decreases from 0.5 to 0.15



Simulations: $\beta = 0$, $s_m = 0.5$, s_w decreases from 0.5 to 0.15



Simulations: $\beta = 1$, $s_m = 0.5$, s_w decreases from 0.5 to 0.15



Conclusions

these results provide a novel explanation of that how lower search costs on a dating market can result in a smaller number of pairs

Further steps:

- prove monotonicity
- more general preferences
- different familiarity graphs
- similar questions...

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